Jet and Rocket Propulsion

AE4451

LECTURE 36

Sedina Tsikata – April 10, 2023

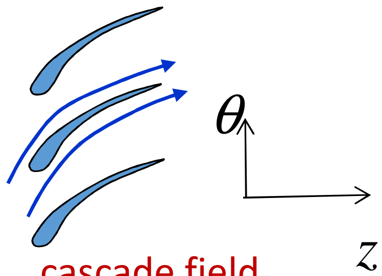
Overview

- what we saw in Lecture 35
  - turbomachinery introduction
    - classifications, compressor types and components
    - Euler turbomachinery equations
    - flowfields, coordinate system and reference frame

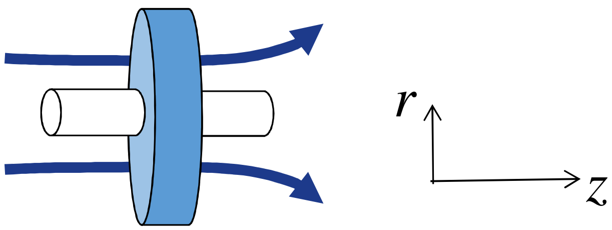
- today
  - axial flow compressors:
    - cascade flow analysis
    - velocity triangles
    - key parameters and calculations

Analysis: a closer look at the flowfields

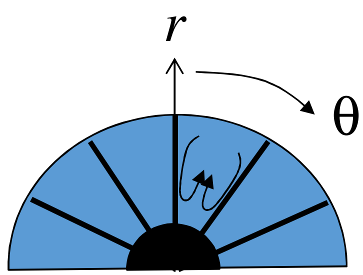
- last lecture, we briefly saw 3 two-dimensional flowfield types



cascade field,
also called
the blade-to-blade field



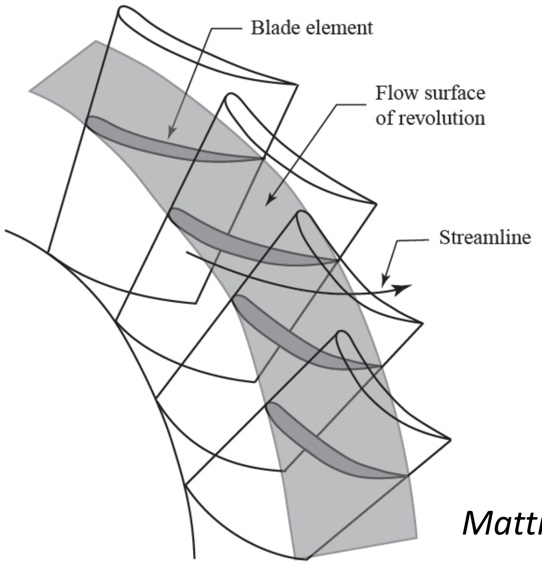
through-flow field



secondary field

- the complete 3D flow field ~ sum of the
  3 less complex 2D flowfields

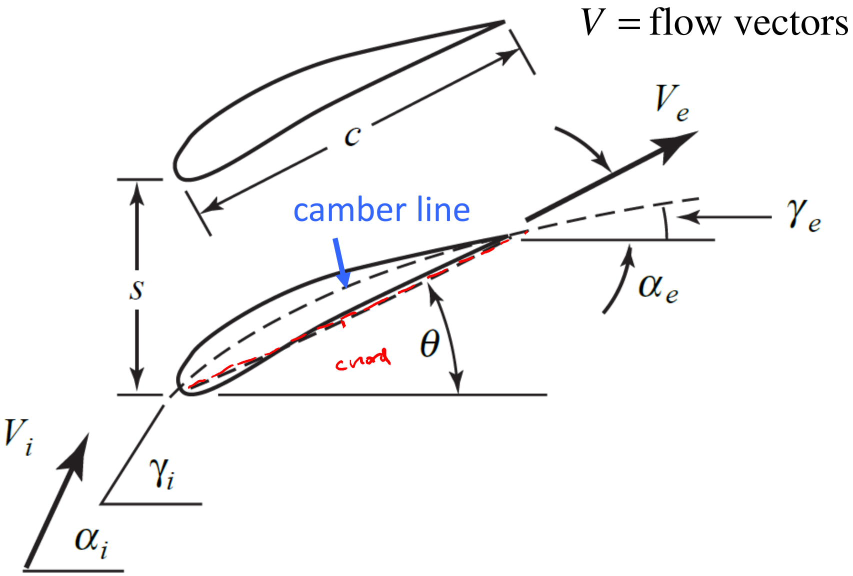
- cascade field
  - considers flow along stream surfaces (s),
    tangentially through blade rows

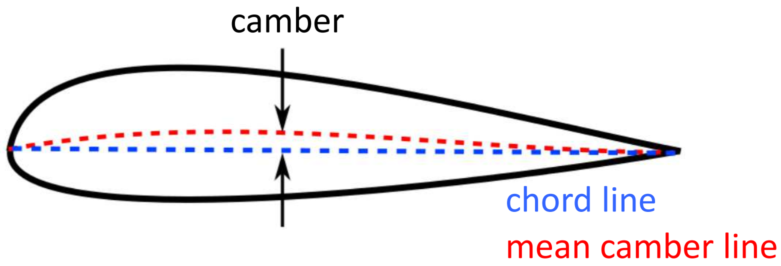


Mattingly

Compressor cascade airfoil nomenclature

- the compressor blade has an airfoil geometry and some specific nomenclature





solidity $\sigma = \frac{c}{s} = \frac{\text{airfoil chord}}{\text{airfoil spacing}}$ usually ~ 1

turning angle $\alpha_i - \alpha_e$ shows how much flow has been turned

airfoil camber angle $\gamma_i - \gamma_e$ change in camber line angle wrt horizontal

incidence angle $\alpha_i - \gamma_i$ difference between flow incident angle and initial camber angle

exit deviation $\alpha_e - \gamma_e = \delta_c$ shows how much exiting flow deviates from camber line

stagger angle θ angle between chord line and horizontal

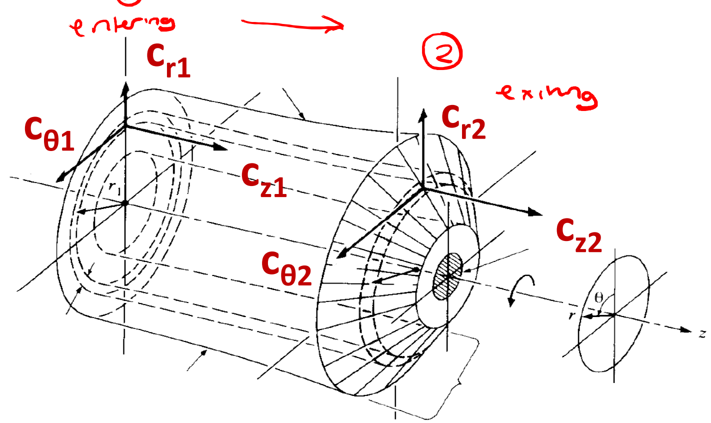
Compressors

Analysis

- for the flow entering and exiting a rotor (from 1 to 2), we found

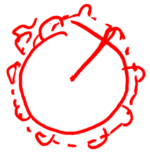
$$\dot{W}/\dot{m} = \Delta(uc_\theta)_{1,2}$$ Euler work equation

control surface (encloses rotor row)



Hill and Peterson

- at the pitch line, and assuming a constant pitch line radius ($r_1 \approx r_2$), $u_m \equiv U$ (rotor velocity U)



$$\Rightarrow \dot{W}/\dot{m} = U\Delta c_{\theta 1,2}$$

- we find Δc_θ (the change in the azimuthal velocity) by analyzing a blade row in the 2D cascade flow field

- this analysis will require flow angles and velocity triangles to visualize changes between reference frames

Compressors

Analysis: cascade flow angles

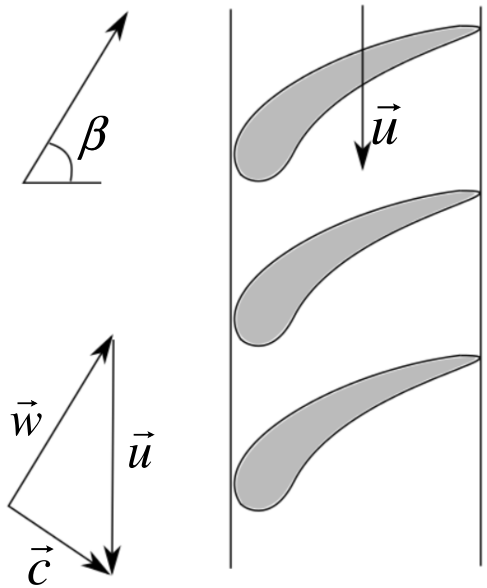
- reminder of relevant reference frames

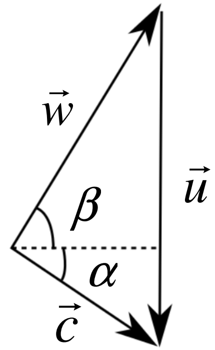
fixed engine reference frame
("absolute velocity"): $\vec{c}$

rotating reference frame
("relative velocity"): $\vec{w}$

blade velocity: $\vec{u}$

$$\Rightarrow \vec{w} = \vec{c} - \vec{u}$$

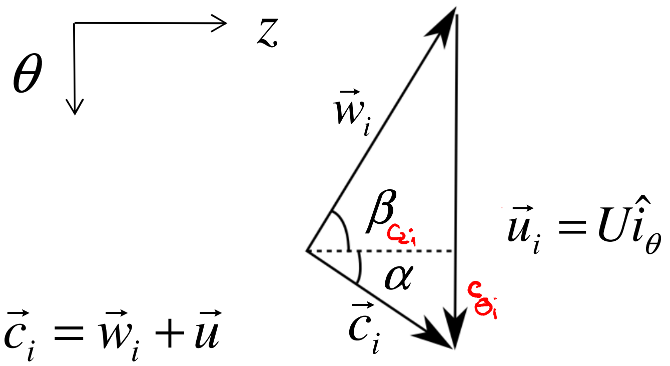




α = absolute flow angle
β = direction of flow relative to rotor

Analysis: velocity triangles

- cascade analysis: flow only in (θ, z); no radial (r) variation



$$\vec{c}_i = \vec{w}_i + \vec{u}$$

α = absolute flow angle
β = direction of flow relative to rotor

- rotor motion in θ direction, so reference frame change has no effect on other directions

$$\vec{w}_{zi} = \vec{c}_{zi}$$

- rotor velocity is constant in cascade flow (like fixed *r*); we can write

$$|u_i| = U \quad \Rightarrow c_{\theta i} - w_{\theta i} = U$$

- can use the velocity triangles to write trigonometric relations:

$$c_{\theta i} = c_i \sin\alpha_i = c_{zi} \tan\alpha_i$$

$$w_{\theta i} = w_i \sin\beta_i = w_{zi} \tan\beta_i$$

$$w_{zi} = w_i \cos\beta_i = c_{zi} = c_i \cos\alpha_i$$

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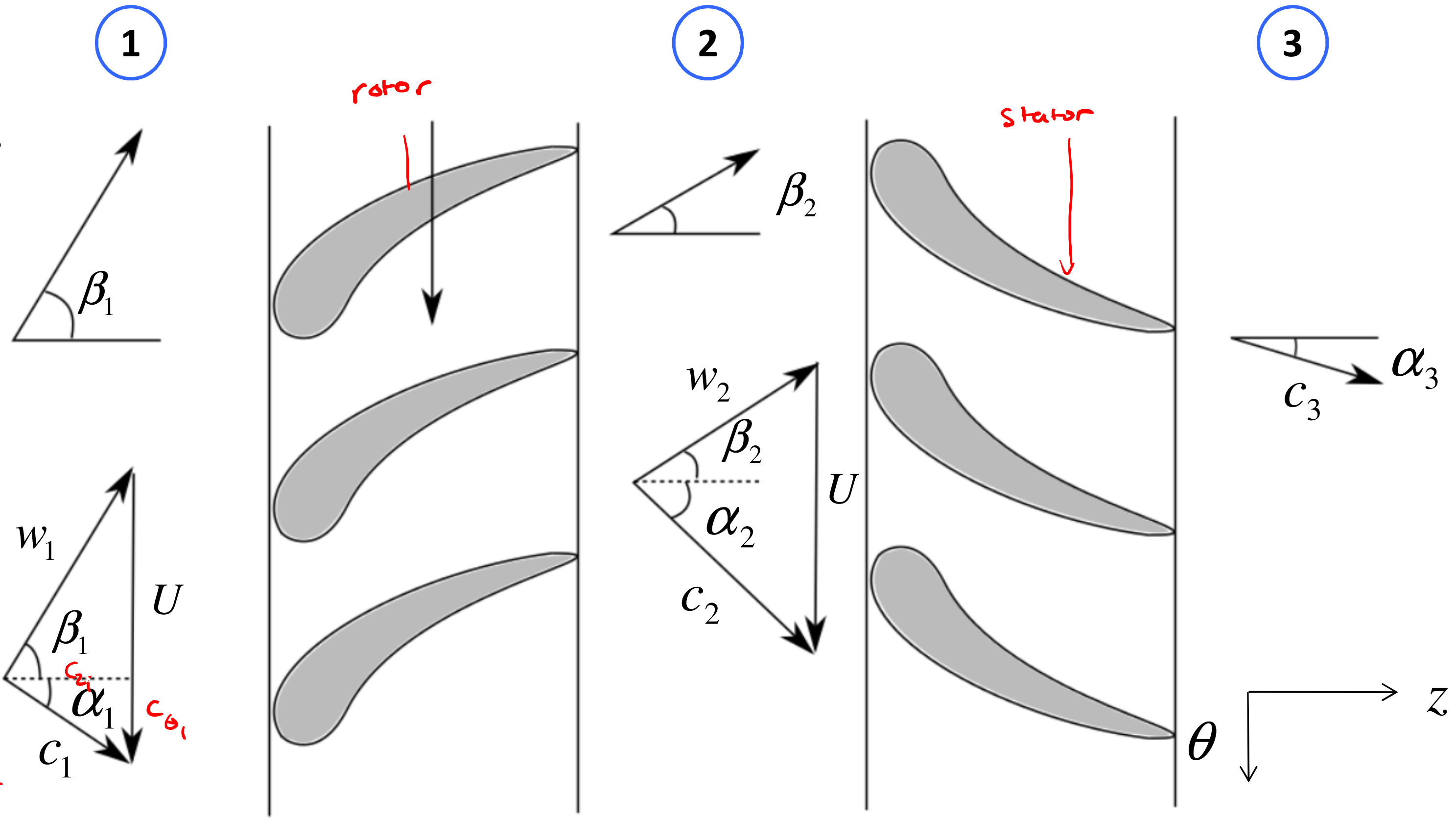
Analysis: velocity triangles

- we can apply this analysis to both the rotor and stator rows to analyze a full stage

- let's define 3 positions:
  - 1 – rotor inlet
  - 2 – between rotor and stator
  - 3 – stator outlet



- using the previous trig relations,

1 – rotor inlet

$$c_{\theta i} = c_i \sin\alpha_i = c_{zi} \tan\alpha_i$$

$$\Rightarrow c_{\theta 1} = c_{z1} \tan\alpha_1$$

$\tan\alpha_1 = \frac{c_{\theta_1}}{c_{z_1}}$

Compressors

Analysis: velocity triangles

(a) $c_{\theta i} = c_i \sin\alpha_i = c_{zi} \tan\alpha_i$

(b) $w_{\theta i} = w_i \sin\beta_i = w_{zi} \tan\beta_i$

(c) $w_{zi} = w_i \cos\beta_i = c_{zi} = c_i \cos\alpha_i$

- using the previous trig relations,

2 – between rotor and stator

$$\vec{w}_{zi} = \vec{c}_{zi} \quad \Rightarrow \vec{w}_{z2} = \vec{c}_{z2}$$

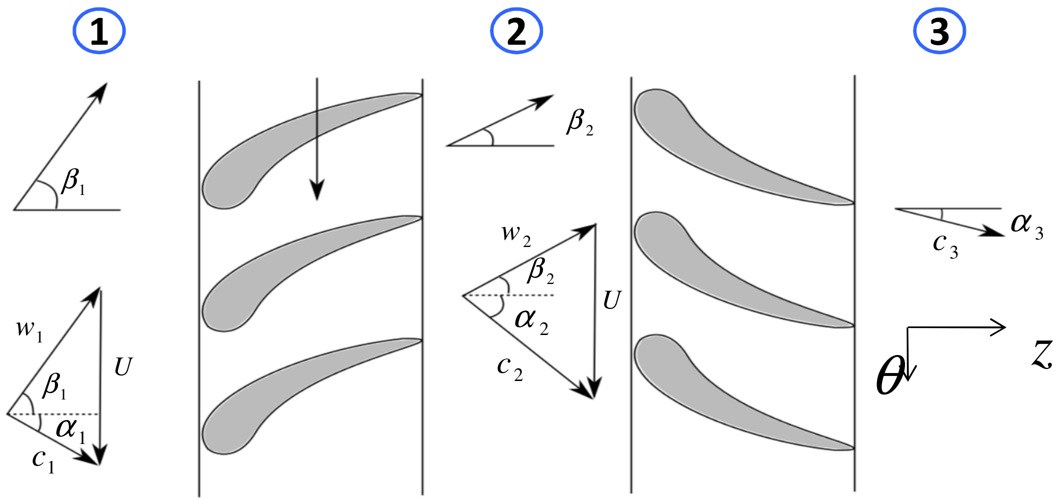
$$w_{\theta i} = w_i \sin\beta_i = w_{zi} \tan\beta_i$$

$$\Rightarrow w_{\theta 2} = c_{z2} \tan\beta_2$$

$$c_{\theta i} - w_{\theta i} = U$$

$$\Rightarrow c_{\theta 2} = U + w_{\theta 2}$$

$$\Rightarrow c_{\theta 2} = U + c_{z2} \tan\beta_2$$



$$\frac{c_{\theta 2}}{c_{z2}} = \frac{U}{c_{z2}} + \tan\beta_2$$

using the form $c_{\theta i} = c_i \sin\alpha_i = c_{zi} \tan\alpha_i$ (a)

$$\Rightarrow \tan\alpha_2 = \frac{U}{c_{z2}} + \tan\beta_2$$

Compressors

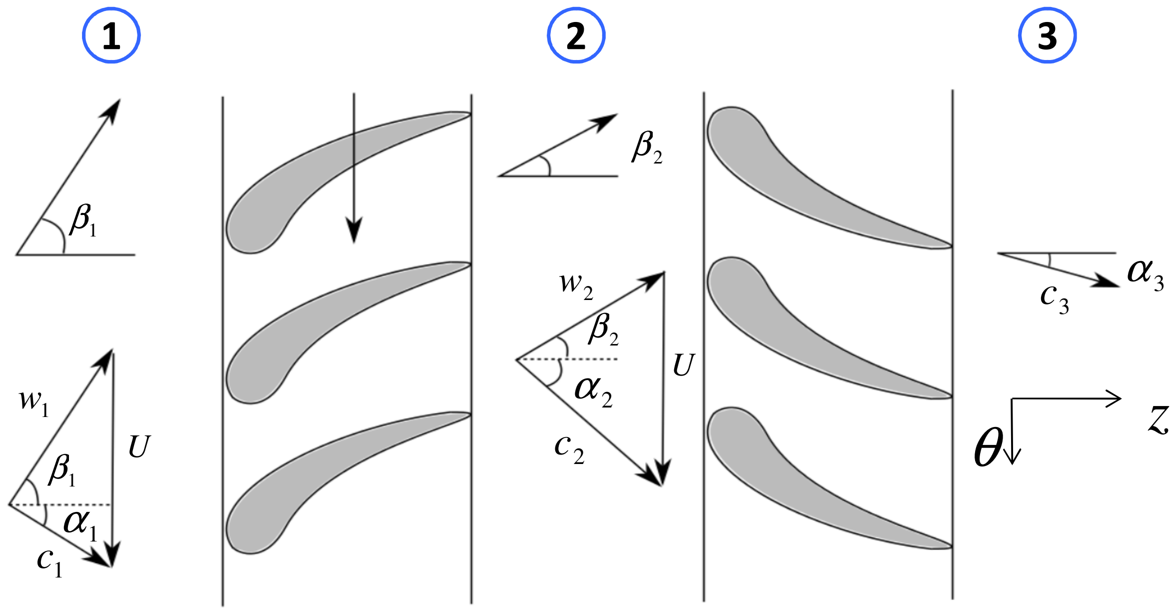
Analysis: velocity triangles

- using the previous trig relations,

3 – at stator outlet

$$c_{\theta i} = c_i \sin \alpha_i = c_{zi} \tan \alpha_i$$

$$c_{\theta 3} = c_{z3} \tan \alpha_3$$



Compressors

Stage characteristics of axial compressors

- we would like to know how compressor performance is affected by changes in operating conditions
  - start by analyzing single stage of compressor

<u>rotor (1 - 2)</u>

$$\dot{W}_R = \dot{m}U\left(c_{\theta_2} - c_{\theta_1}\right) = \dot{m}\left(h_{o2} - h_{o1}\right)$$

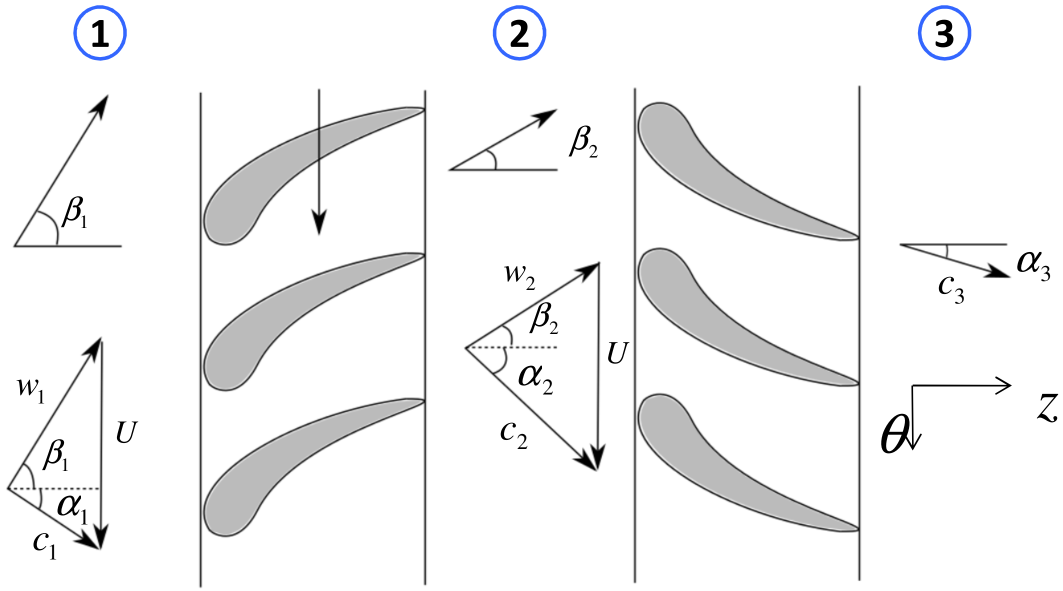
Euler work equations

$$U\Delta c_{\theta_{1,2}} = \Delta h_{o_{1,2}}$$

<u>stator (2 - 3)</u>

$$\dot{W}_S = 0 \Rightarrow h_{o3} = h_{o2}$$

so, overall for the stage: $\Delta h_{o_{1,3}} = \Delta h_{o_{1,2}} = U\Delta c_{\theta_{1,2}}$



from our prior velocity triangle analysis, using expressions for $c_{\theta 1}$ and $c_{\theta 2}$

$$\Delta c_{\theta_{1,2}} = U + c_{z_2}\tan\beta_2 - c_{z_1}\tan\alpha_1$$

if c_z = constant, then: $\Delta c_{\theta_{1,2}} = U + c_z\left(\tan\beta_2 - \tan\alpha_1\right)$

Compressors

Compressor stage pressure ratio

- consider expressions used for adiabatic compressor, thermally and calorically perfect gas

- we can apply these to an individual compressor stage, i.e.,

$$\frac{p_{o3}}{p_{o1}} = \left[1+\eta_{st}\left(\frac{T_{o3}}{T_{o1}}-1\right)\right]^{\gamma/\gamma-1} = \left[1+\eta_{st}\left(\frac{T_{o3}-T_{o1}}{T_{o1}}\right)\right]^{\gamma/\gamma-1}$$

stage efficiency

$$= \left[1+\eta_{st}\frac{\Delta h_{o_{1,3}}}{c_p T_{o1}}\right]^{\gamma/\gamma-1}$$

$h = c_p T \rightarrow \Delta h = c_p \Delta T$

$$= \left[1+\eta_{st}\frac{(\gamma-1)U^2}{\gamma R T_{o1}}\frac{\Delta h_{o_{1,3}}}{U^2}\right]^{\gamma/\gamma-1}$$

from using the forms $\left.\begin{matrix}\frac{c_p}{c_v}=\gamma \\ c_p - c_v = R\end{matrix}\right\} c_p = \frac{\gamma R}{\gamma-1}$

$$\frac{\Delta h_{o_{1,3}}}{U^2} = \frac{\Delta c_{\theta_{1,2}}}{U}$$

from previous stage analysis $\Delta h_{o_{1,3}} = \Delta h_{o_{1,2}} = U\Delta c_{\theta_{1,2}}$

compressor stage pressure ratio

$$\Rightarrow \mathrm{Pr}_{stage} \equiv \frac{p_{o3}}{p_{o1}} = \left[1+\eta_{st}(\gamma-1)\frac{U^2}{\gamma R T_{o1}}\frac{\Delta c_{\theta_{1,2}}}{U}\right]^{\gamma/\gamma-1}$$

Compressors

Compressor stage pressure ratio

$$\mathrm{Pr}_{stage} \equiv \frac{p_{o3}}{p_{o1}} = \left[1 + \eta_{st}(\gamma-1)\frac{U^2}{\gamma R T_{o1}}\frac{\Delta c_{\theta_{1,2}}}{U}\right]^{\gamma/\gamma-1}$$

compressor stage pressure ratio

- Pr_{stage} depends on:

1. stage loading coefficient, Ψ $\psi = \frac{\Delta c_{\theta_{1,2}}}{U}$

2. blade Mach number $\frac{U}{\sqrt{\gamma R T_{o1}}}$
   - rotor blade speed U = rΩ
   - stage inlet stagnation temperature = T_{o1}

3. stage efficiency η_{st}

- stage loading coefficient $\psi = \frac{\Delta h_{o_{1,3}}}{U^2} = \frac{\Delta c_{\theta_{1,2}}}{U}$

interpretation: $\frac{\text{work/mass}}{\text{rotor kinetic energy}} \sim \frac{\text{swirl increase}}{\text{rotor velocity}}$

for typical axial compressor designs: Ψ ~ 0.3 - 0.5

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Compressor stage pressure ratio

- flow coefficient

  - we showed $\psi = \frac{\Delta c_{\theta_{1,2}}}{U} = 1 + \frac{c_z}{U}\left(\tan\beta_2 - \tan\alpha_1\right)$

  - substituting a flow coefficient ϕ: $\frac{c_z}{U} \equiv \phi$

$$\psi = 1 + \phi\left(\tan\beta_2 - \tan\alpha_1\right)$$

for typical axial compressor designs: ϕ ~ 0.4 - 0.8

Compressors

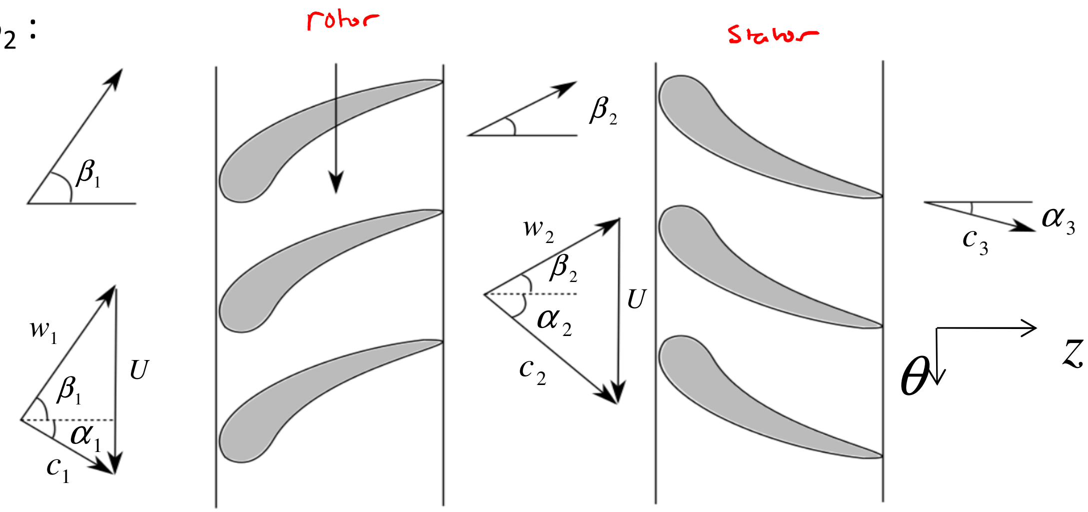
Influence of blade design

flow coefficient

stage loading coefficient $$\psi = 1 + \phi(\tan\beta_2 - \tan\alpha_1)$$

stage loading dependent on:

- ϕ : decrease flow coefficient, increase stage loading $$\frac{c_z}{U} \equiv \phi$$
- inlet flow angle α_1 - from previous stage or inlet guide vanes (IGVs)
- flow angle leaving rotor blade in rotor reference frame β_2 : depends on blade design angle



Compressors

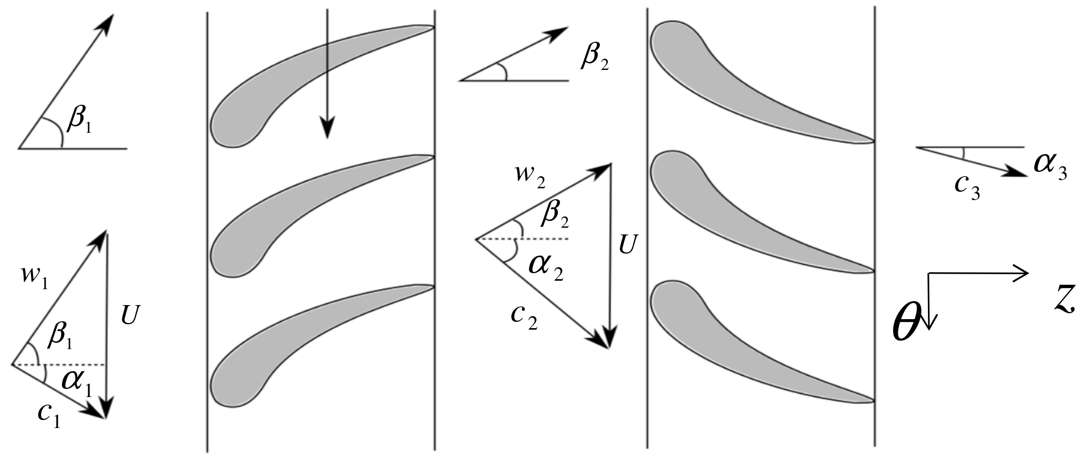
Example of compressor design and resulting parameters

Consider an axial air compressor stage with a design point created for:

- 8,000 rpm rotor with 0.30 m mean design radius
- 100 m/s axial velocity (constant through stage),
  36.4 m/s inlet tangential velocity
- 50° rotor blade trailing edge angle (wrt to axis and opposite to rotation), and 20° stator blade's trailing edge angle
- 95% estimated adiabatic stage efficiency @ design point
- 298 K inlet temperature, γ = 1.4

Determine at design point:

1. rotor speed (m/s)
2. stage flow coefficient
3. stage loading coefficient
4. required power per flowrate (kJ/kg)
5. stagnation pressure ratio across stage



assume - calorically/thermally perfect gas, zero radial velocity

Compressors

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Example of compressor design and resulting parameters

1. find rotor speed in m/s

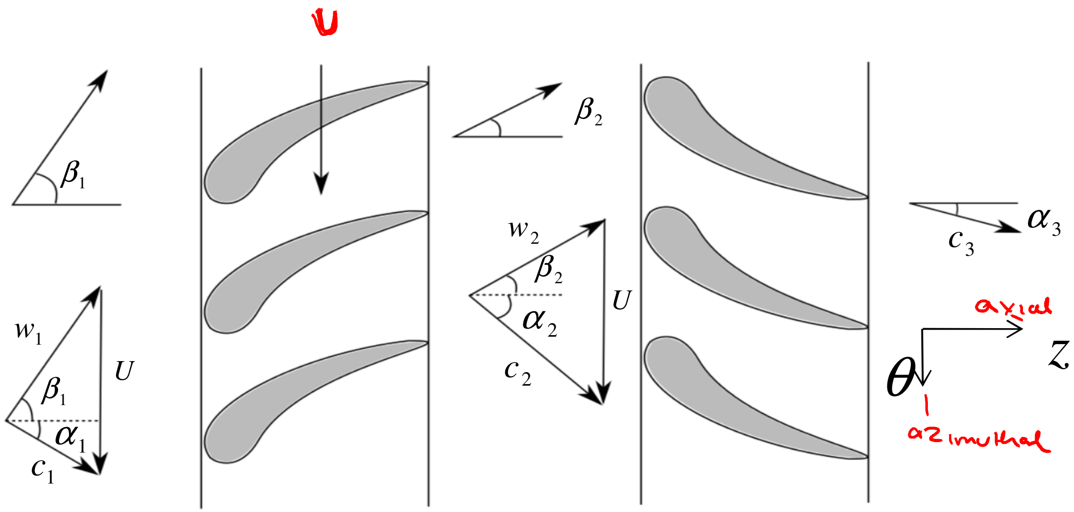
- 8,000 rpm rotor with 0.30 m mean design radius

$$\Omega = N\left(\frac{rev}{min}\right)\left(\frac{min}{60sec}\right)\left(\frac{2\pi}{rev}\right)$$

$$= (N_{rpm}\,\pi/30)$$

$$U = \Omega r = (N_{rpm}\,\pi/30)r$$

$$= 8000 \times \frac{\pi}{30} \times 0.3 = 251\,\text{m/s}$$



2. find stage flow coefficient

$$\phi = \frac{c_z}{U}$$

- 100 m/s axial velocity specified, constant through the stage: this is c_z

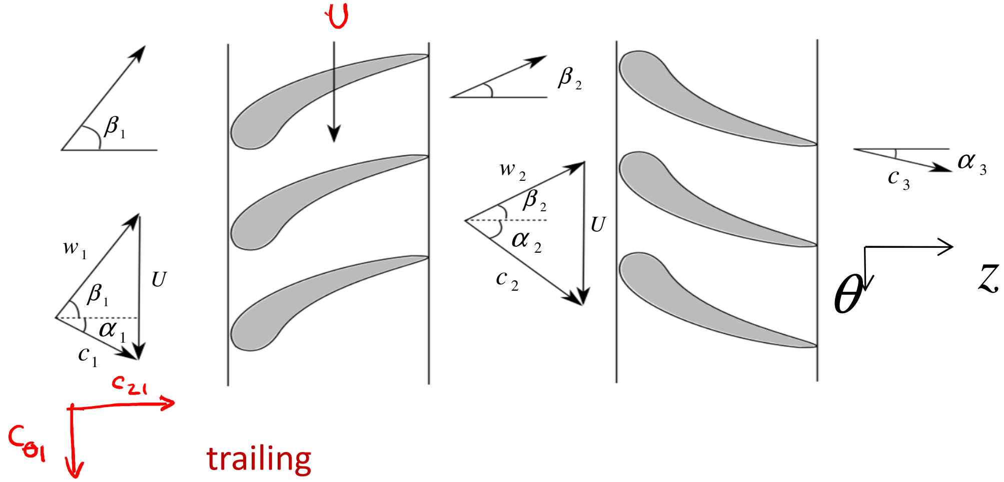
$$\Rightarrow \phi = \frac{100}{251} = 0.398$$

Example of compressor design and resulting parameters

3. find stage loading coefficient

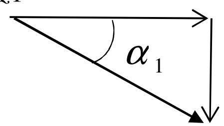
$$\psi = 1 + \phi\left(\tan\beta_2 - \tan\alpha_1\right)$$

specified:
- 36.4 m/s inlet tangential velocity
- 50° rotor blade trailing edge angle (wrt to axis and opposite to rotation)



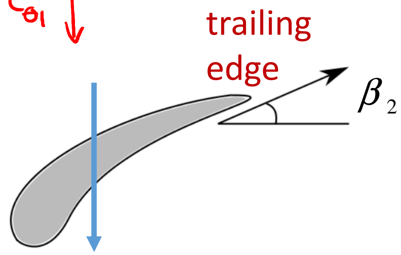
$$\tan\alpha_1 = \frac{c_{\theta 1}}{c_{z1}}$$

$$c_{z1} = 36.4 \text{ m/s}$$



$$c_{\theta 1} = 100 \text{ m/s}$$

$$\tan\alpha_1 = \frac{36.4}{100} \Rightarrow \alpha_1 = 20°$$

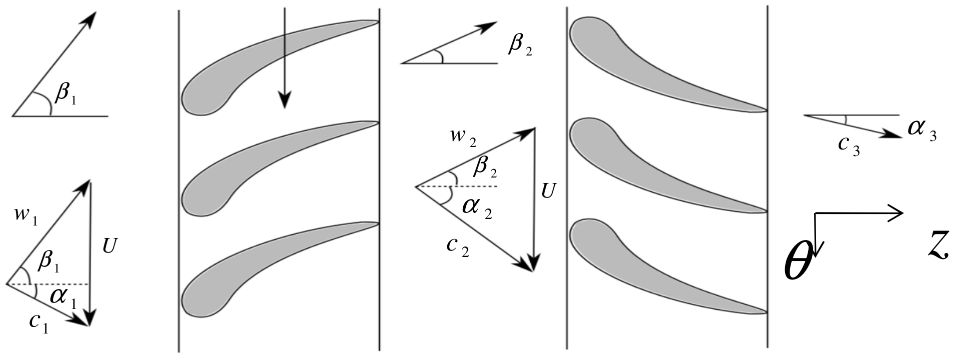


$$\Rightarrow \Psi = 1 + 0.398\left(\tan(-50) - \frac{36.4}{100}\right) = 0.381$$

Compressors

Example of compressor design and resulting parameters

4. find required power per flowrate (kJ/kg)



$$\dot{W}/\dot{m}_a = h_{o3} - h_{o1} = U^2 \psi$$

$$\frac{\dot{W}}{\dot{m}_a} = (251)^2 0.381 = 24.1 \frac{kJ}{kg}$$

5. find stagnation pressure ratio across the stage $\frac{P_{03}}{P_{01}}$

specified:
- 95% estimated adiabatic stage efficiency @ design point
- 298 K inlet temperature, γ = 1.4

$$Pr_{st} = \left[1 + \eta_{st}(\gamma - 1)\frac{U^2}{\gamma R T_{o1}} \frac{\Delta c_{\theta_{1,2}}}{U}\right]^{\gamma/\gamma-1} = \left[1 + 0.95(1.4-1)\left(\frac{251\, m/s}{\sqrt{1.4(288\, J/kgK)298K}}\right)^2 0.381\right]^{3.5}$$

ψ

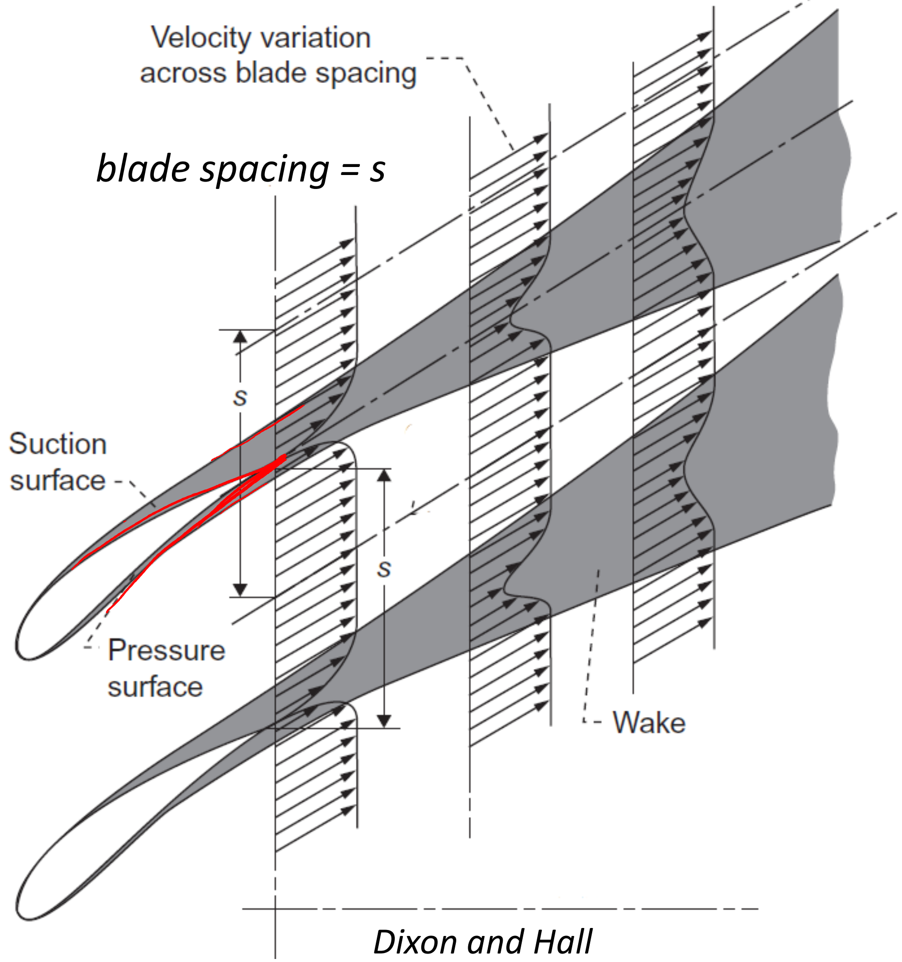
$$\Rightarrow Pr_{st} = 1.3$$

Compressors

Real flow around rotor blades in the compressor

- near the design point, the flow (mostly) follows the blade
  - this is an idealization
- deviation from the blade arises through inviscid and viscous effects
- flow unable to follow the blade angle precisely (underturned) and leaves trailing edge at slightly different angle to the blade exit angle
- growth of the boundary layers on the suction and pressure surfaces of the blades = source of cascade losses
- boundary layers combine at the blade trailing edge where they form the blade wake
- at high Mach number: losses due to shock waves and shock boundary layer interaction at the blade surfaces

wake formation



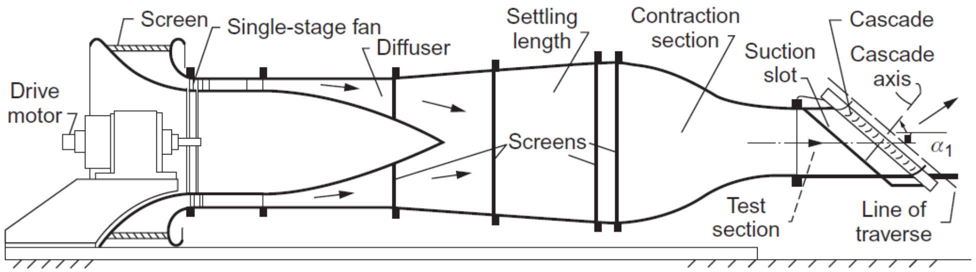
Dixon and Hall

Compressors

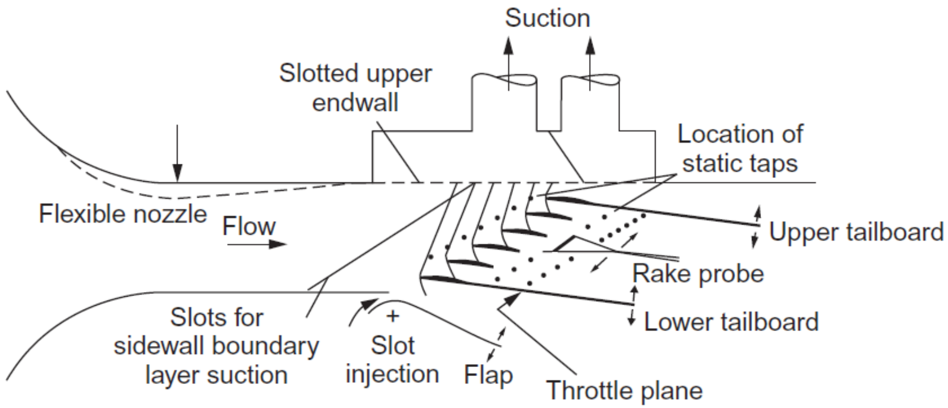
Analysis: cascade field measurements

- experimental testing performed in cascade tunnels
- in axial flow turbomachines: M = 0.1 – 2.5

- deviation and loss for a cascade are measured/ computed at range of conditions
- checks of performance and good off-design behavior



low-speed, continuous running cascade tunnel



cascade tunnel for transonic, moderate supersonic flows

- slotted upper equipped for suction, allowing operation in transonic regimes
- flexible upper wall to change of geometry (formation of con-di nozzle to reach supersonic speeds)

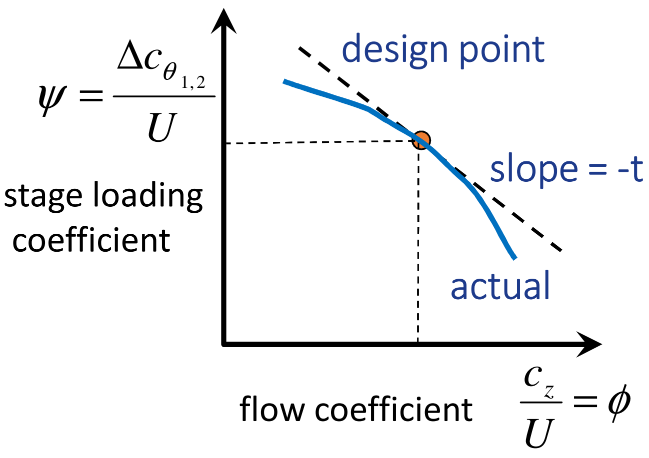
Dixon and Hall

Compressors

Compressor stage off-design performance

- when the compressor operates at mass flow rates or rotational speeds that are away from the design point
  blades subject to incidence variations

- for stable (off-design) operation, important to have tolerance (usually 5% variation of incidence without stalling)



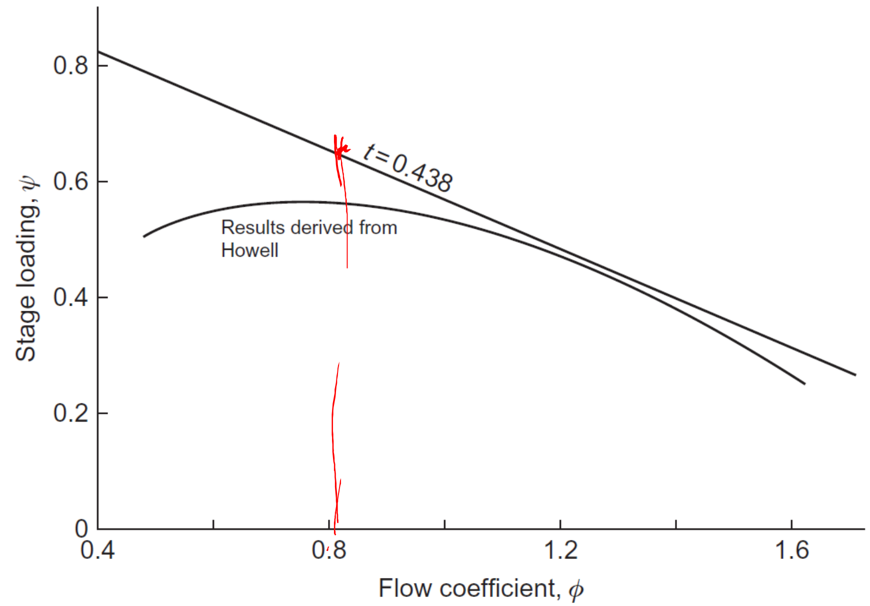
$$\psi = \frac{\Delta c_{\theta_{1,2}}}{U} = 1 + \frac{c_z}{U}(\tan\beta_2 - \tan\alpha_1) = 1 - t \times \frac{c_z}{U}$$

e.g. Howell test results (1945)

design point for compressor stage:

$$\phi = 0.8$$

+ values of angles



Dixon and Hall